

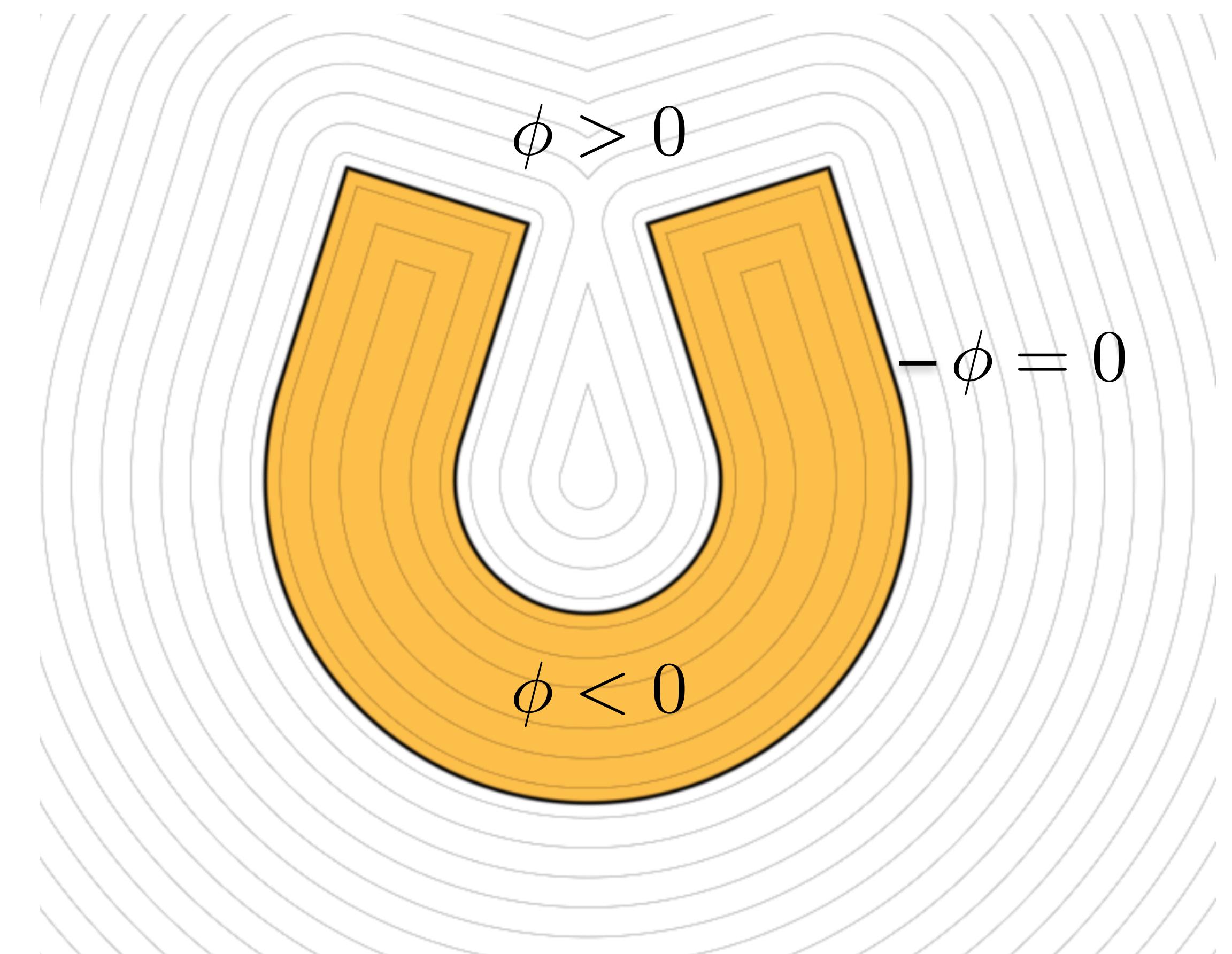
Local Optimization for Robust Signed Distance Field Collision

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QUICK RECAP ON SDFs

- Implicit shape representation:
 - Inside / outside
 - Discrete or analytic
 - GPU friendly
- Explicit surface representation:
 - Triangle mesh
 - Convex mesh



[Quilez 2008]

USING SDFS FOR COLLISION DETECTION

- Contact constraint

$$C(\mathbf{x}) = \phi(\mathbf{x}) - r \geq 0$$

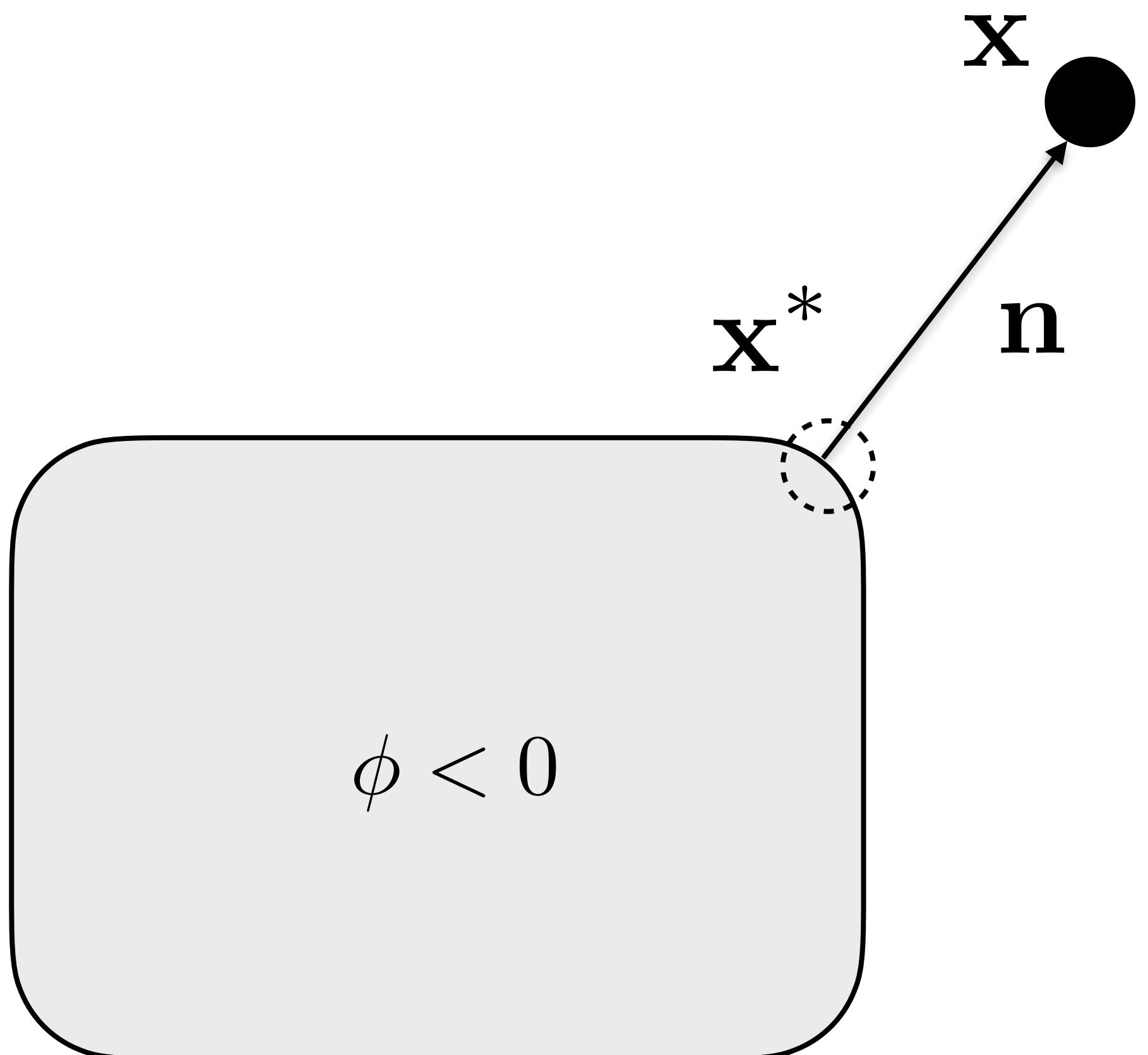
- Normal is given by gradient:

$$\mathbf{n} = \nabla \phi(\mathbf{x})$$

- Closest point is given by:

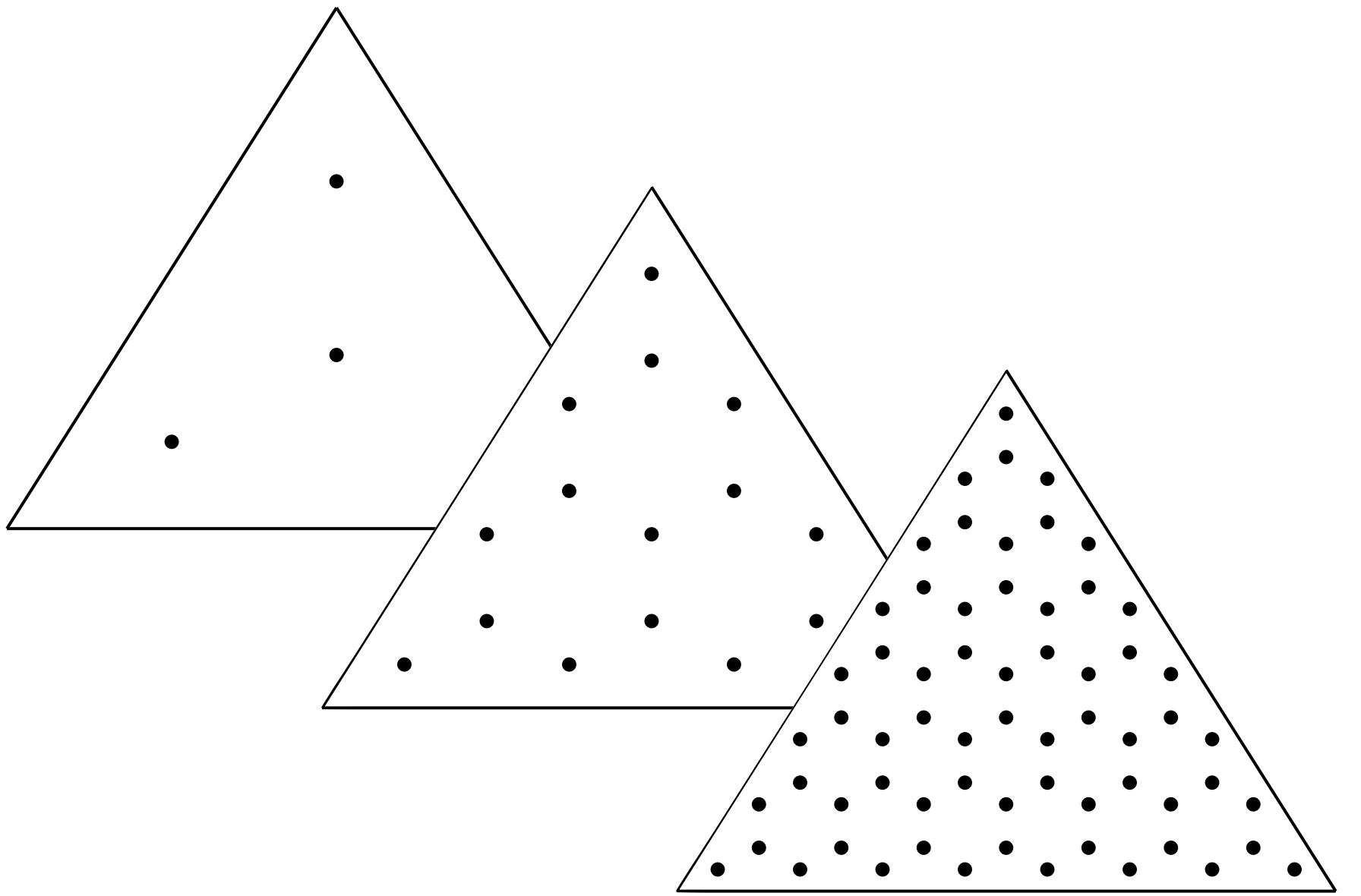
$$\mathbf{x}^* = \mathbf{x} - \nabla \phi(\mathbf{x}) \phi(\mathbf{x})$$

- Not an approximation!



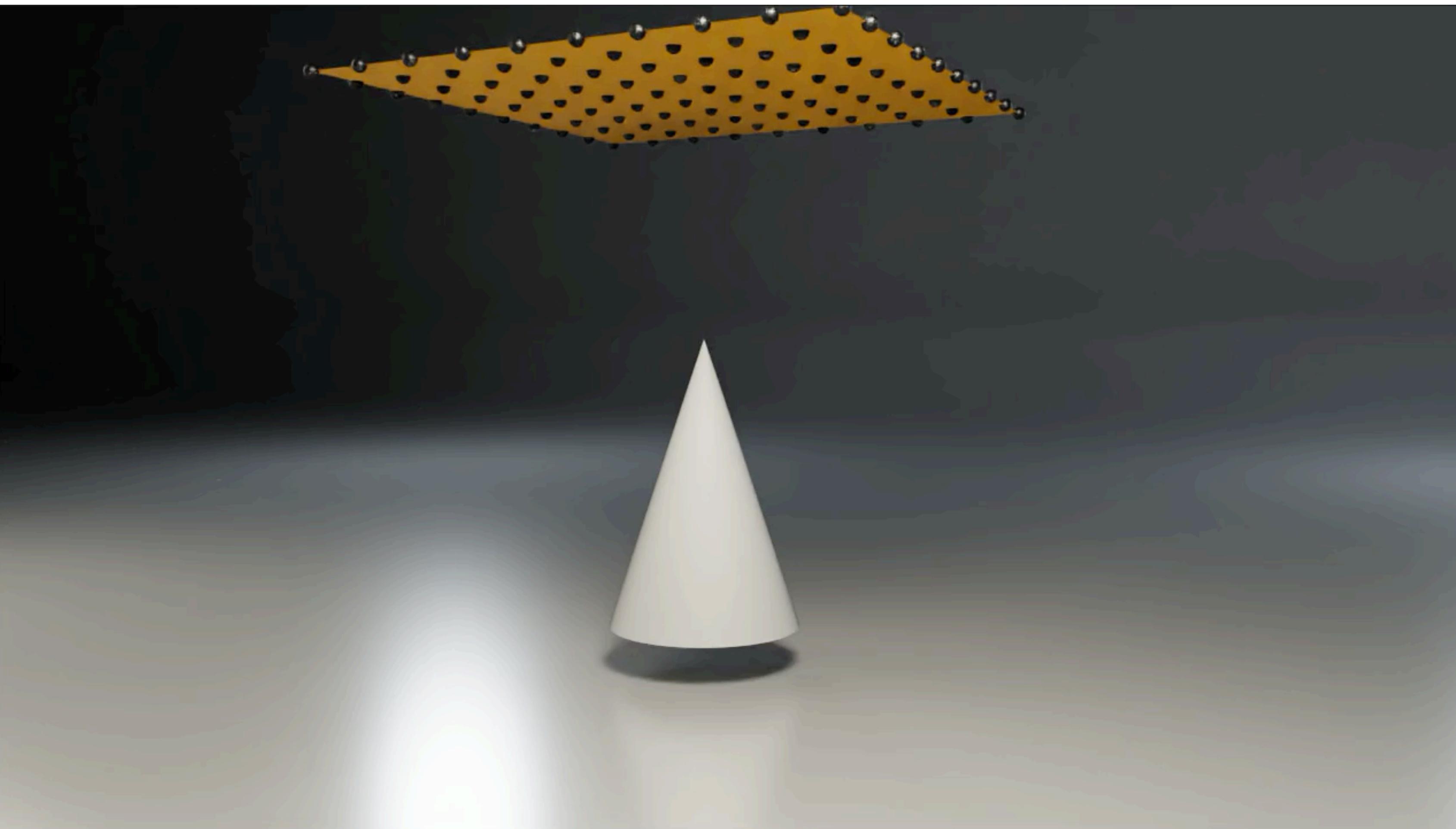
HOW TO HANDLE SURFACE CONTACT

- SDFs are queried at a point in space
- How to query surfaces, e.g.: cloth?
- One approach: **point sample**
- Generate many samples on the surface and consider them all
- **Pros:**
 - Very simple
 - Highly parallel
- **Cons:**
 - Generates lots of contacts
 - Can still miss features

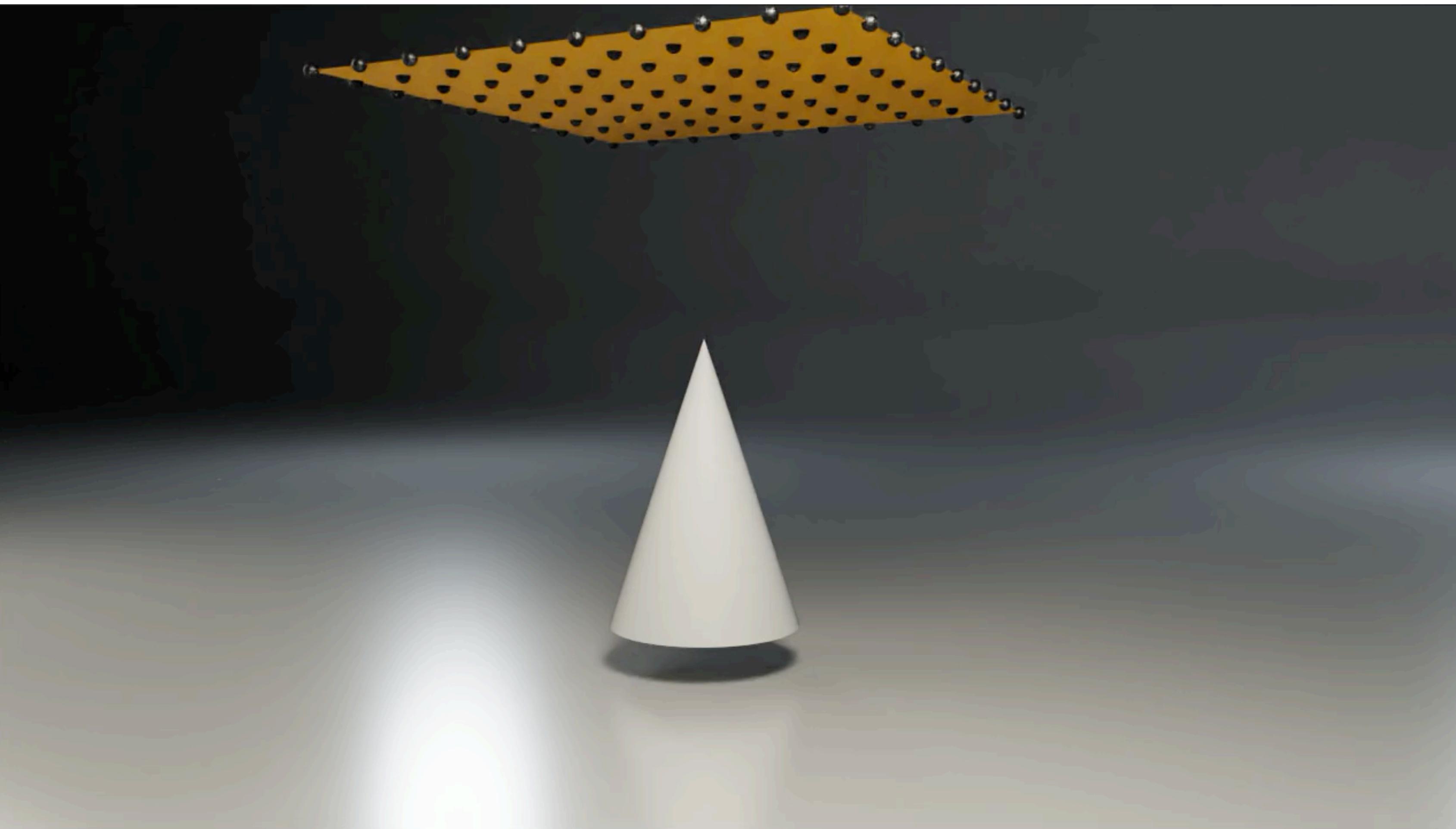


[Basu 2015]

POINT SAMPLING - 1X

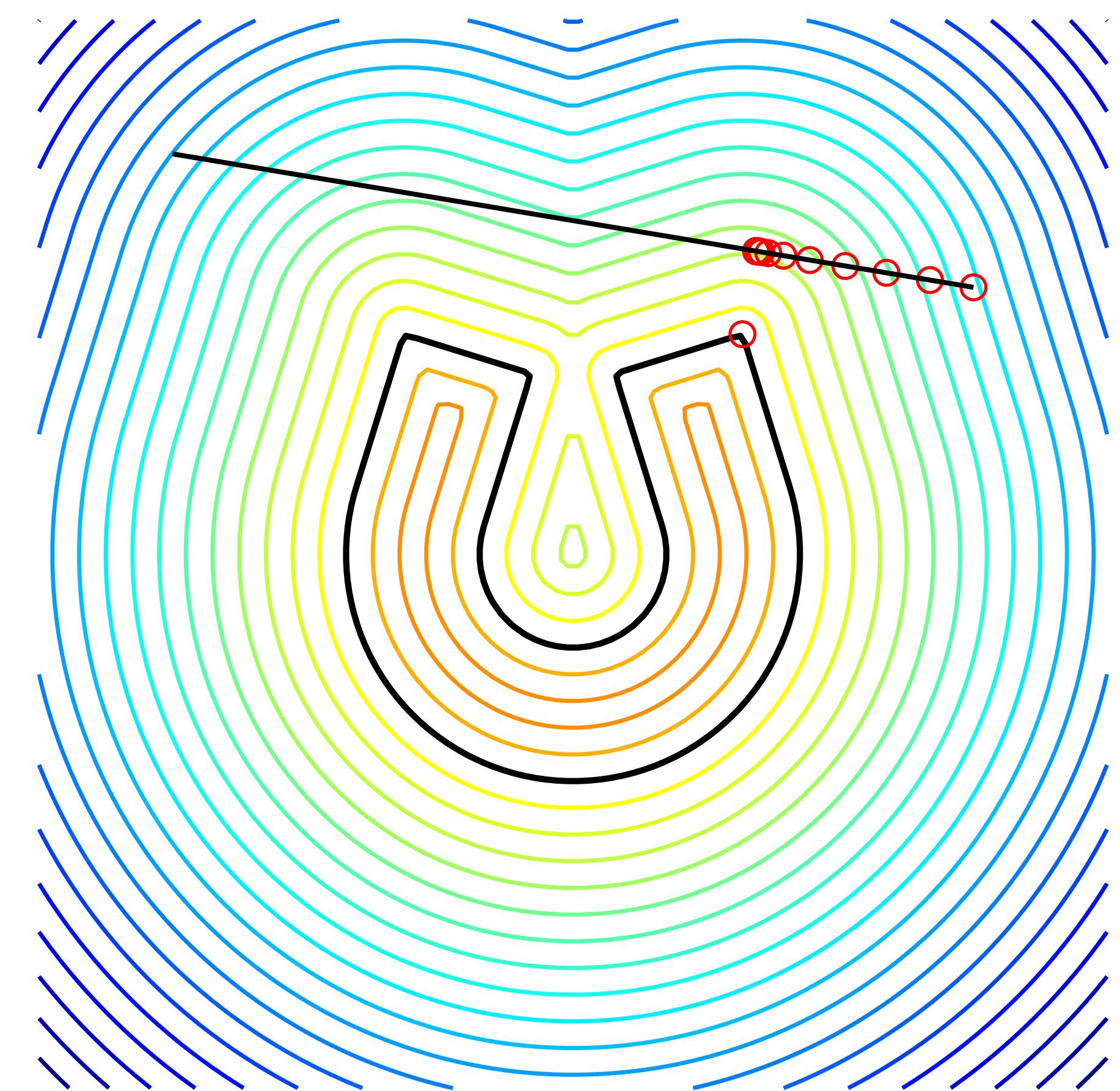


POINT SAMPLING - 64X



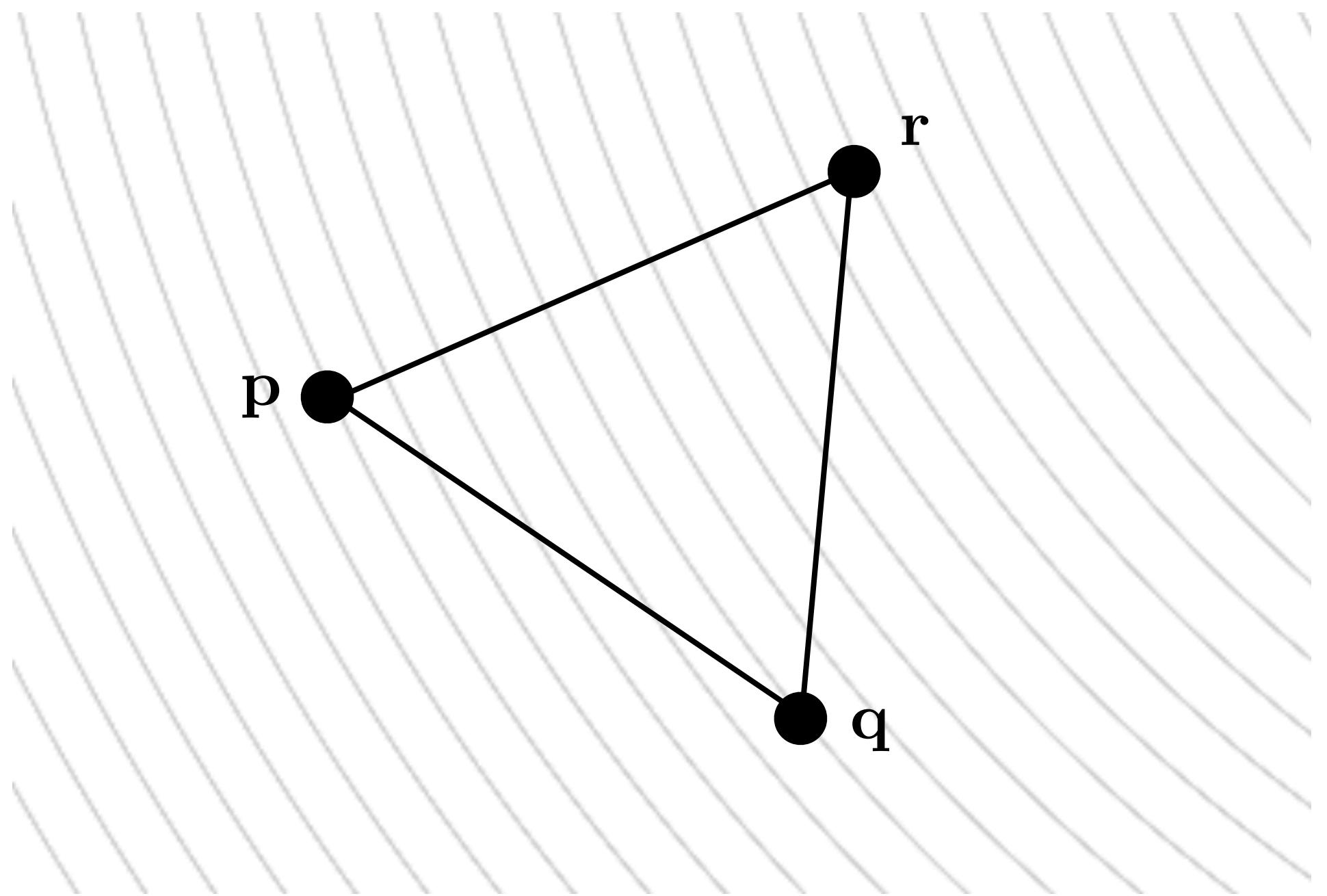
OUR APPROACH

- We propose a local optimization approach to generate **continuous contacts**
- Find closest or deepest point over each mesh element, e.g.: edge/face
- Generate contact at this point



OPTIMIZATION PROBLEM

- Given a convex domain (edge, face)
- Minimize **non-linear** and **non-convex** objective
- In general SDFs may represent arbitrarily complex functions
- Can only hope for a **local minimum**
- For many cases this is acceptable



$$\underset{u,v,w}{\operatorname{argmin}} \quad \phi(u\mathbf{p} + v\mathbf{q} + w\mathbf{r})$$

$$\text{s.t. } u, v, w \geq 0$$

$$u + v + w = 1$$

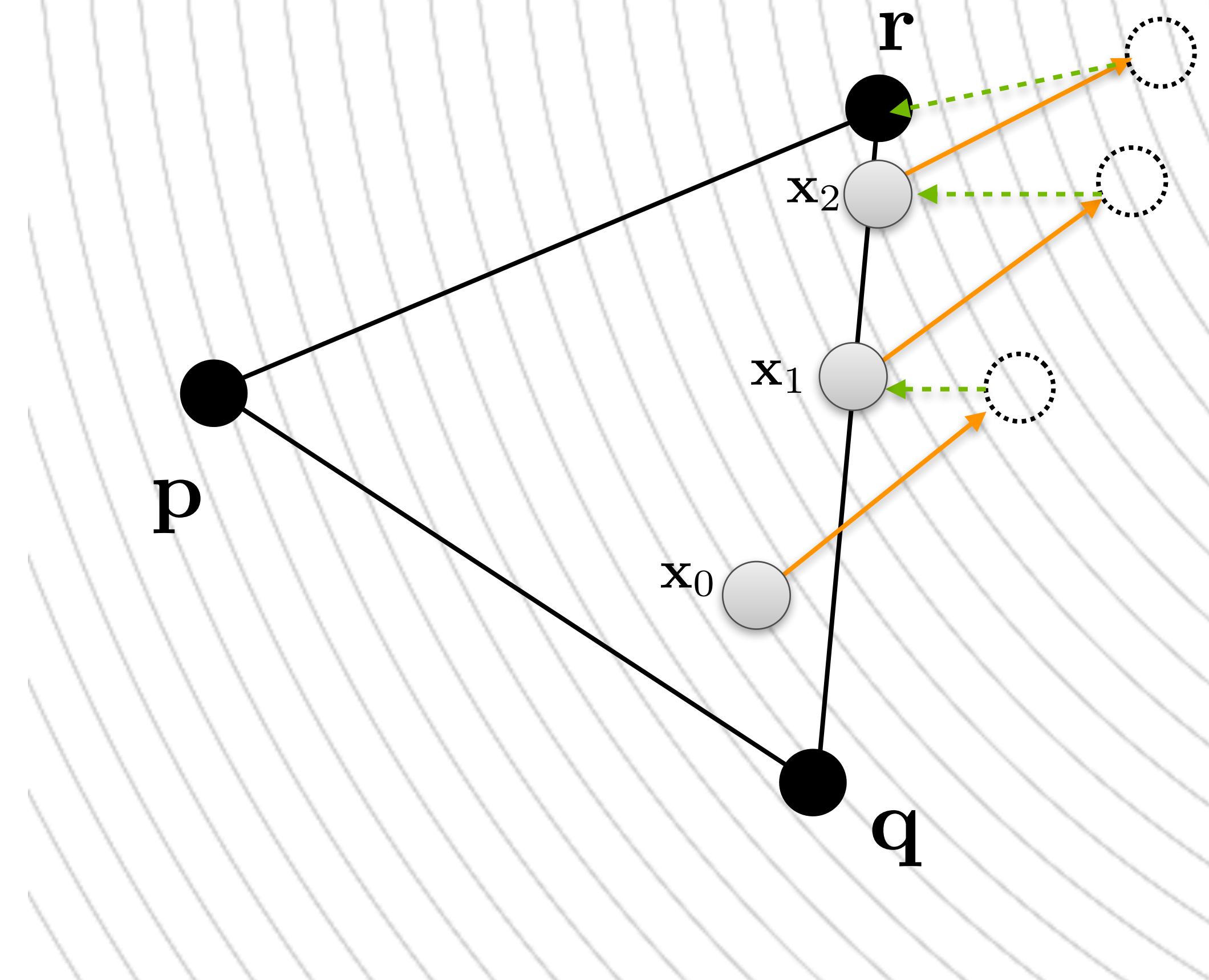
PROJECTED GRADIENT DESCENT

- Simplest possible optimization method
- Move along the SDF gradient
- If we step out of the face then **project back** onto the closest point

$$\mathbf{x}_{i+1} = \mathbb{P}(\mathbf{x}_i - \alpha \nabla \phi)$$

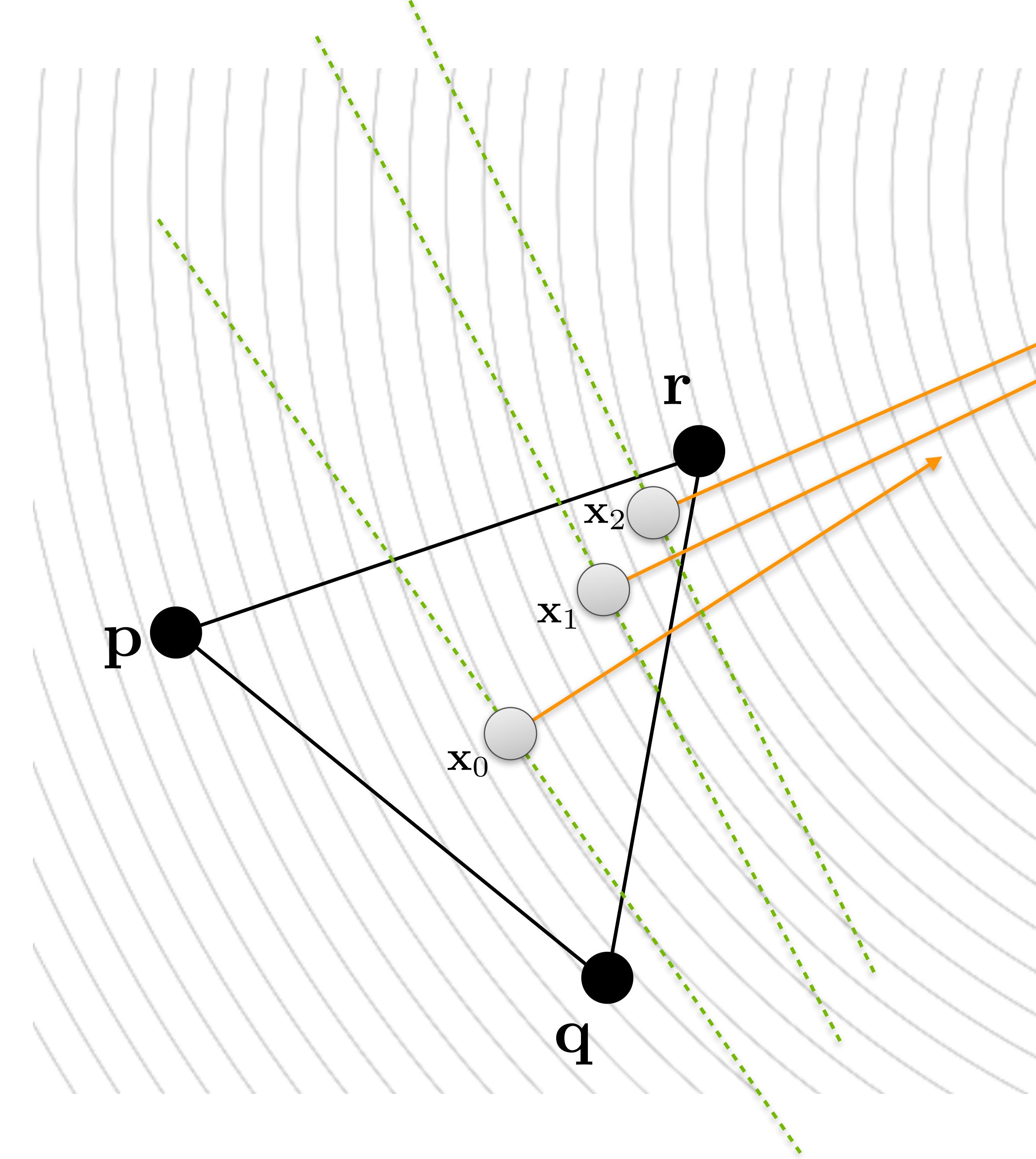
- Requires choosing a step-length to move along (or line search)
- Good news, gradient has **unit magnitude**:

$$\|\nabla \phi(\mathbf{x})\| = 1$$



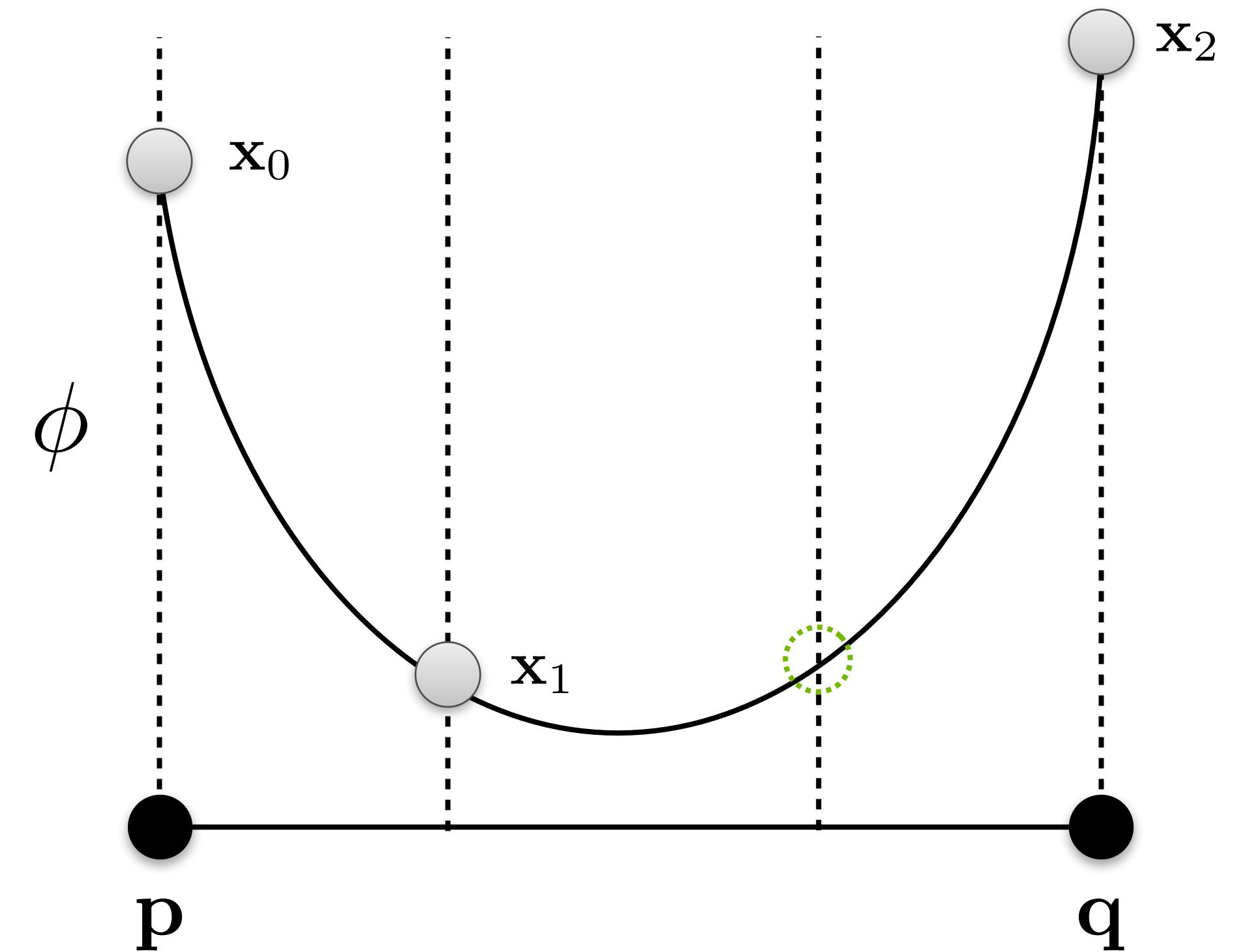
FRANK-WOLFE METHOD

- Convex optimization on a convex domain
- Does not require any projection operation
- Linearize objective and pick best vertex
- Uses a decreasing step size to achieve convergence (no step size)
- [Jaggi 2013]



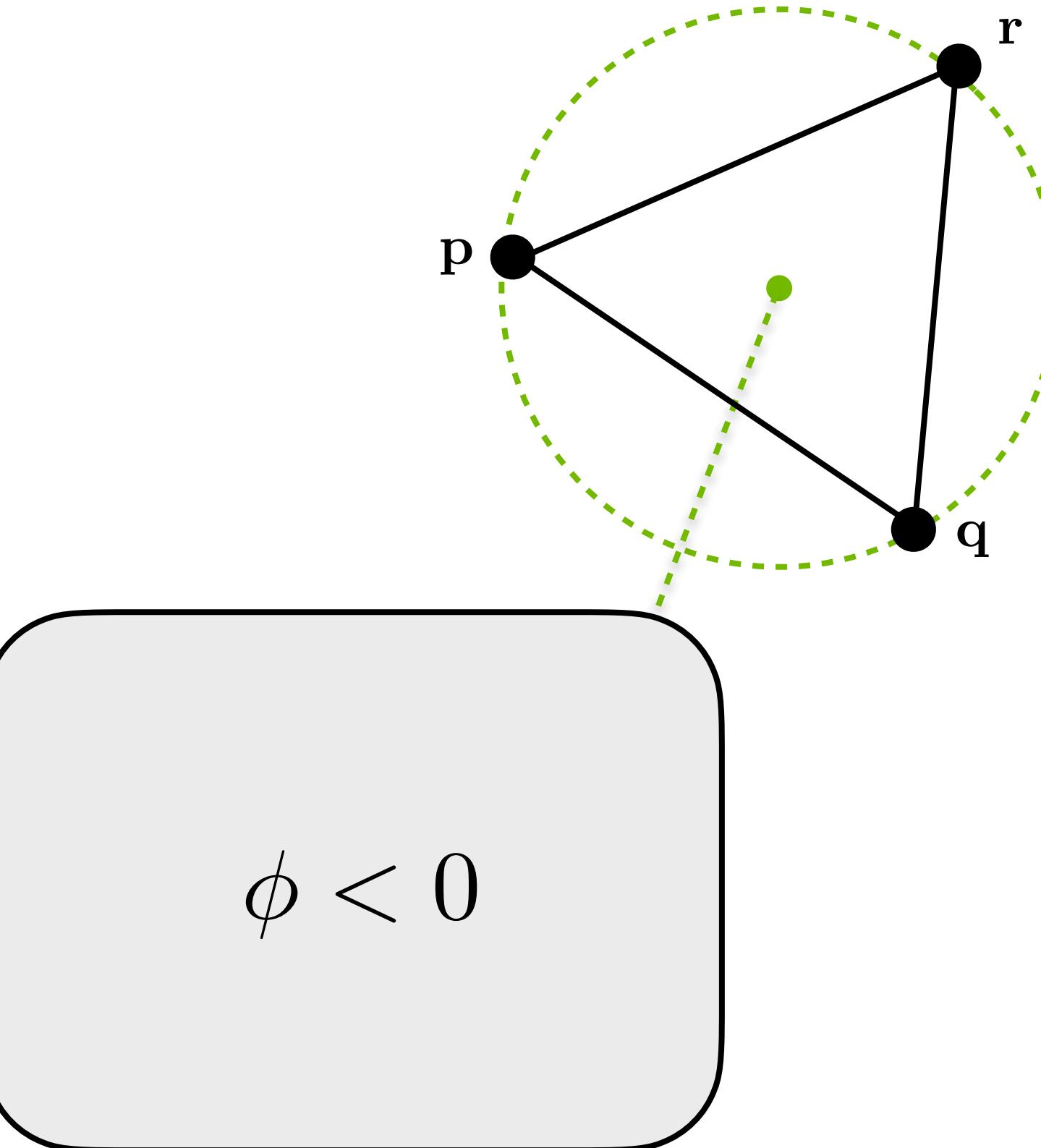
GOLDEN-SECTION SEARCH

- Generalization of bisection (root finding) to optimization problems
- Applicable to 1D problems (i.e.: edges)
- Uses a recurrence based on Fibonacci to define iterates



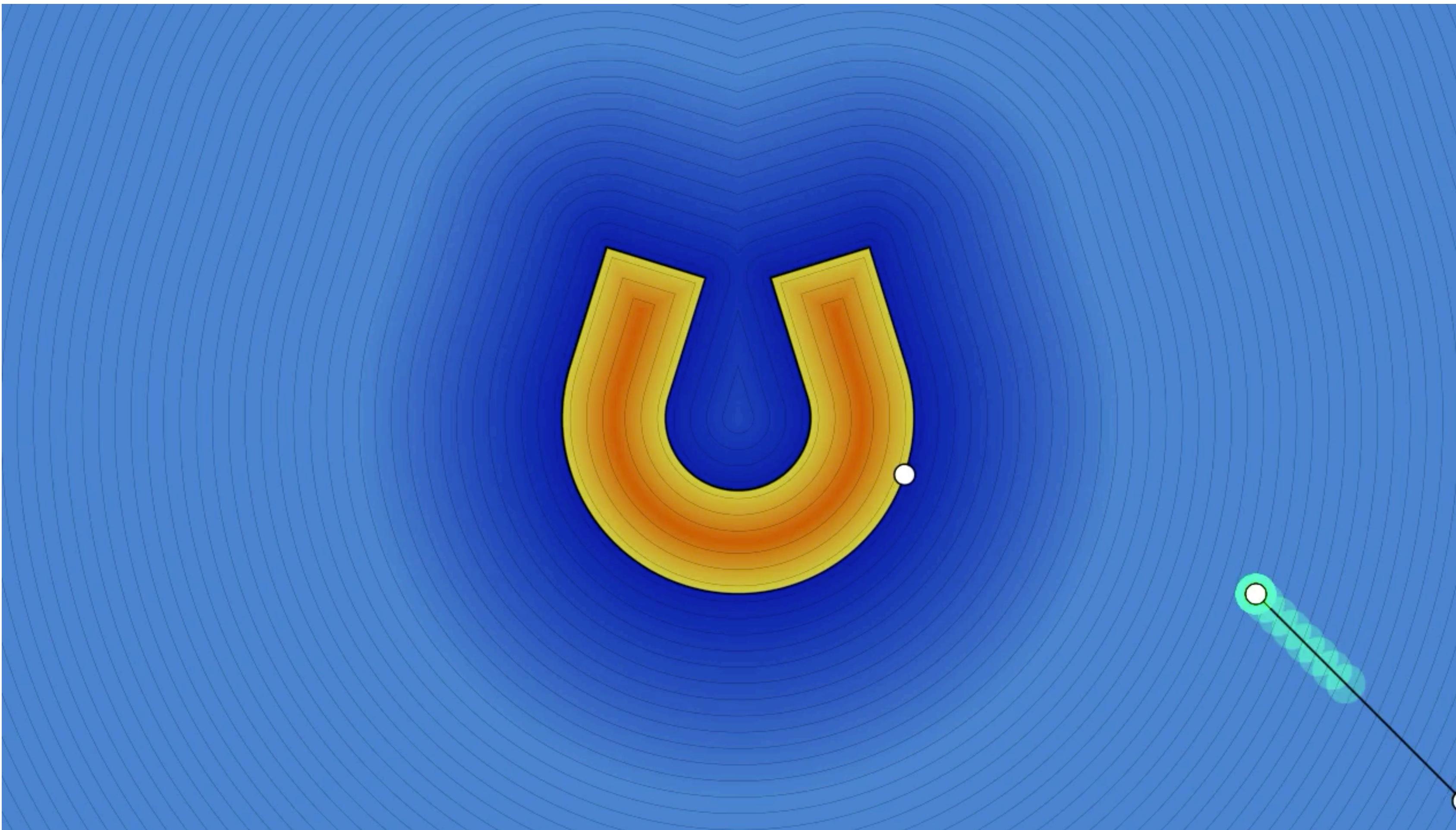
ALGORITHM

- for each element (edge, tri)
 - Test bounding sphere (culling)
 - Pick starting point (midpoint, minimum)
 - while !converged:
 - $x^* = \text{step}(x)$
 - if $\text{sdf}(x^*) < \text{contact_dist}$
 - output contact (x^*)



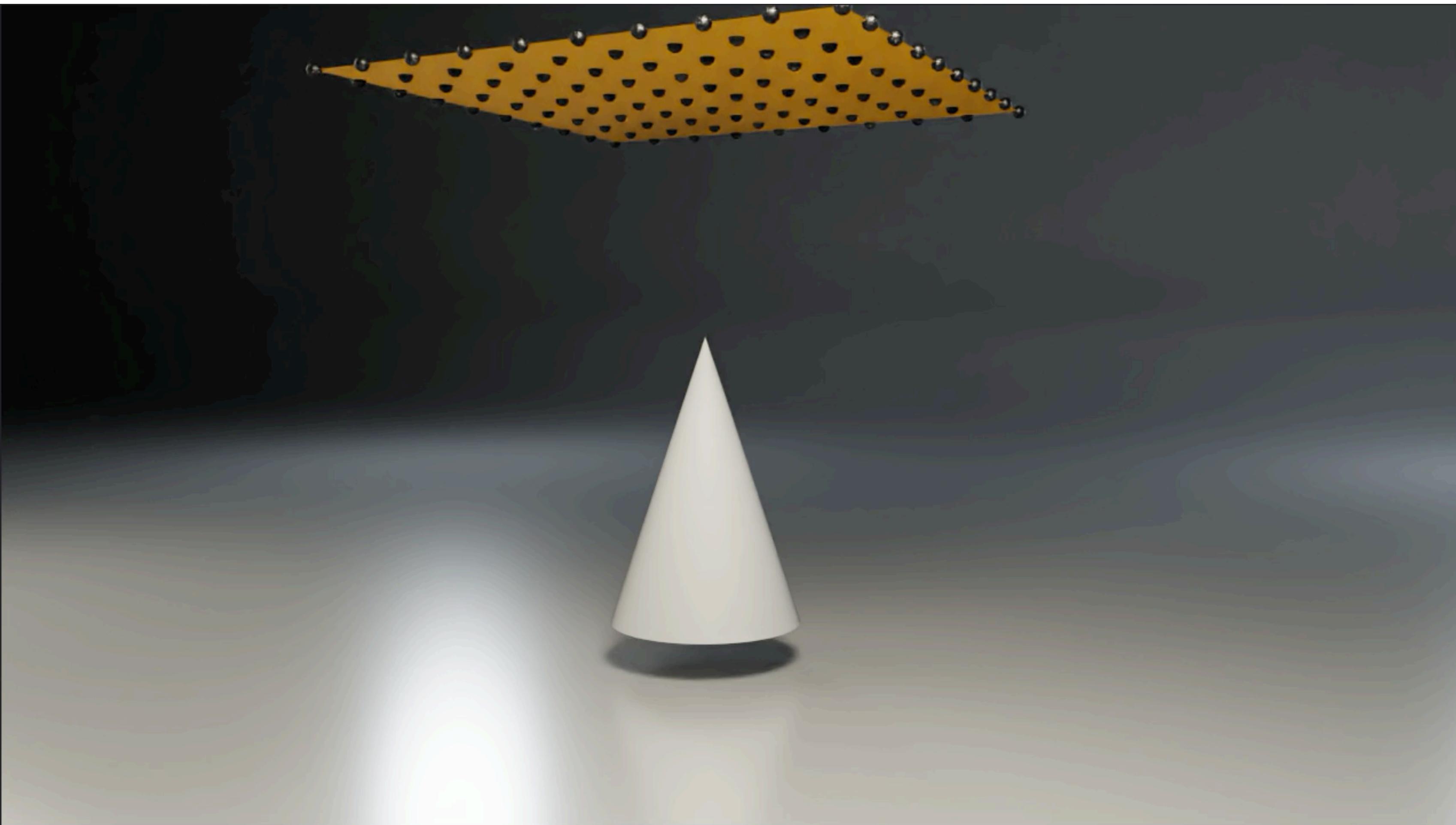
Bounding Sphere Culling

SHADER TOY



<https://www.shadertoy.com/view/wdcGDB>

RESULTS - OUR METHOD



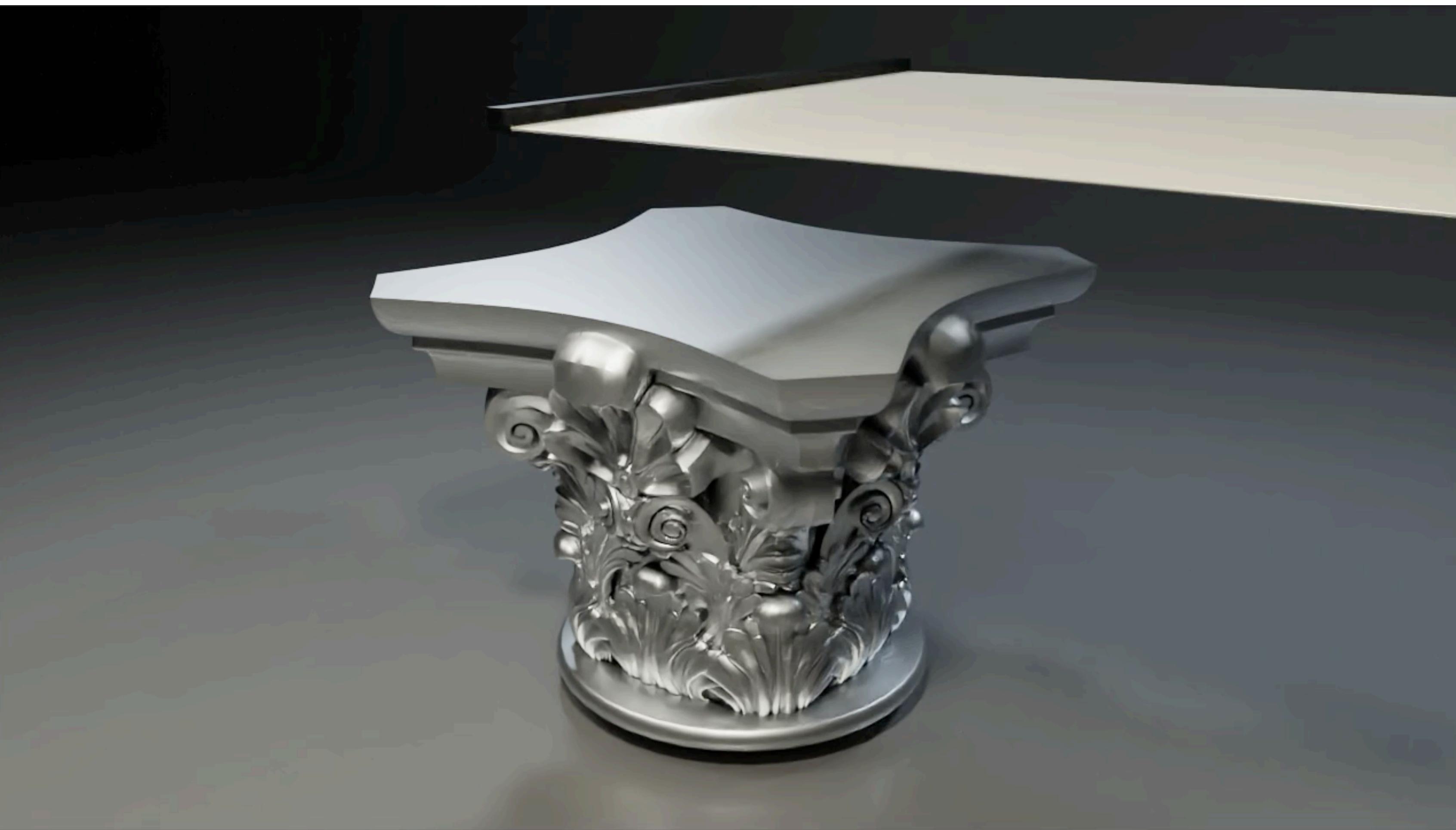
RESULTS - POINT SAMPLED



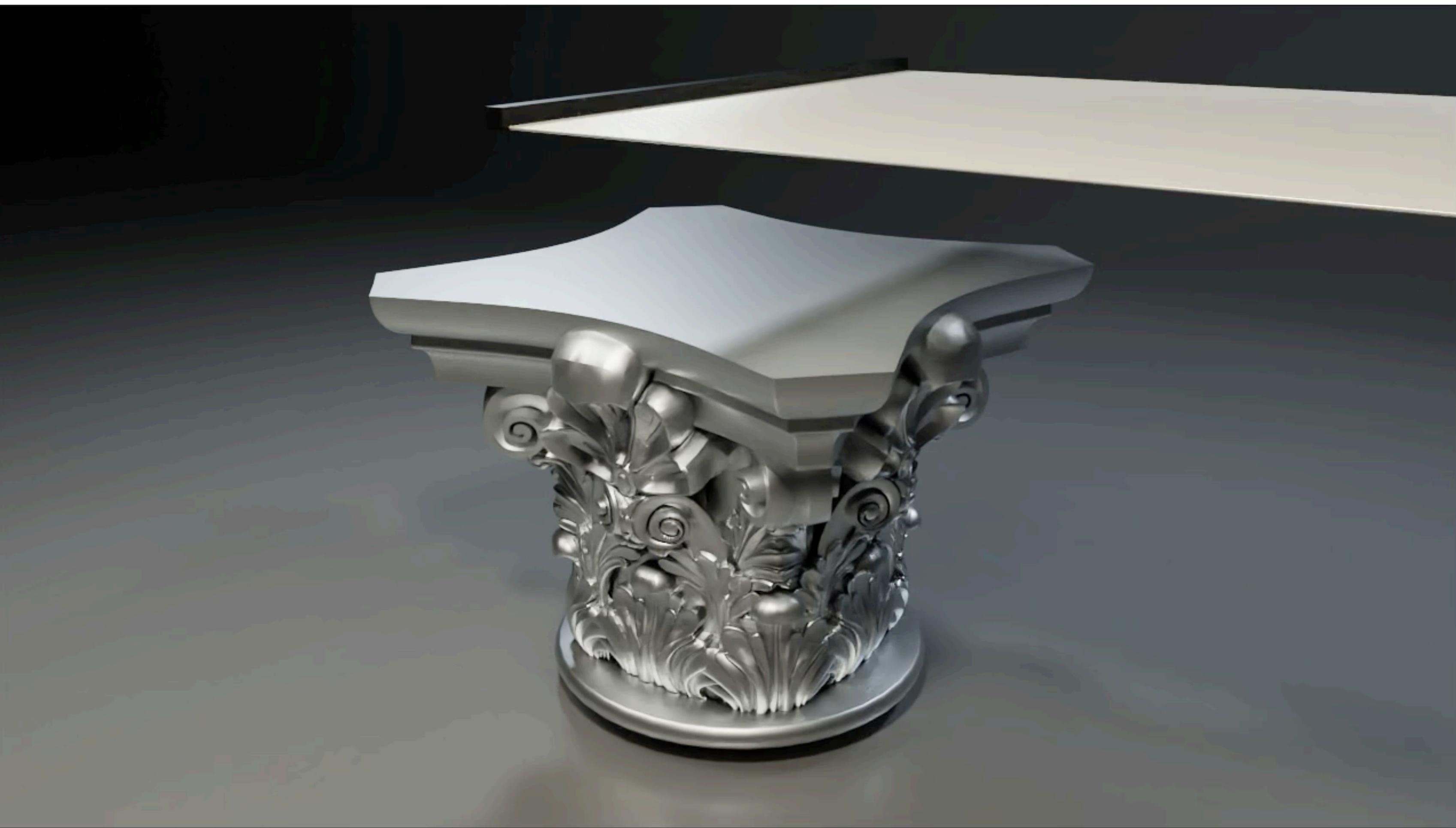
RESULTS - FACE SAMPLED



RESULTS - POINT SAMPLED



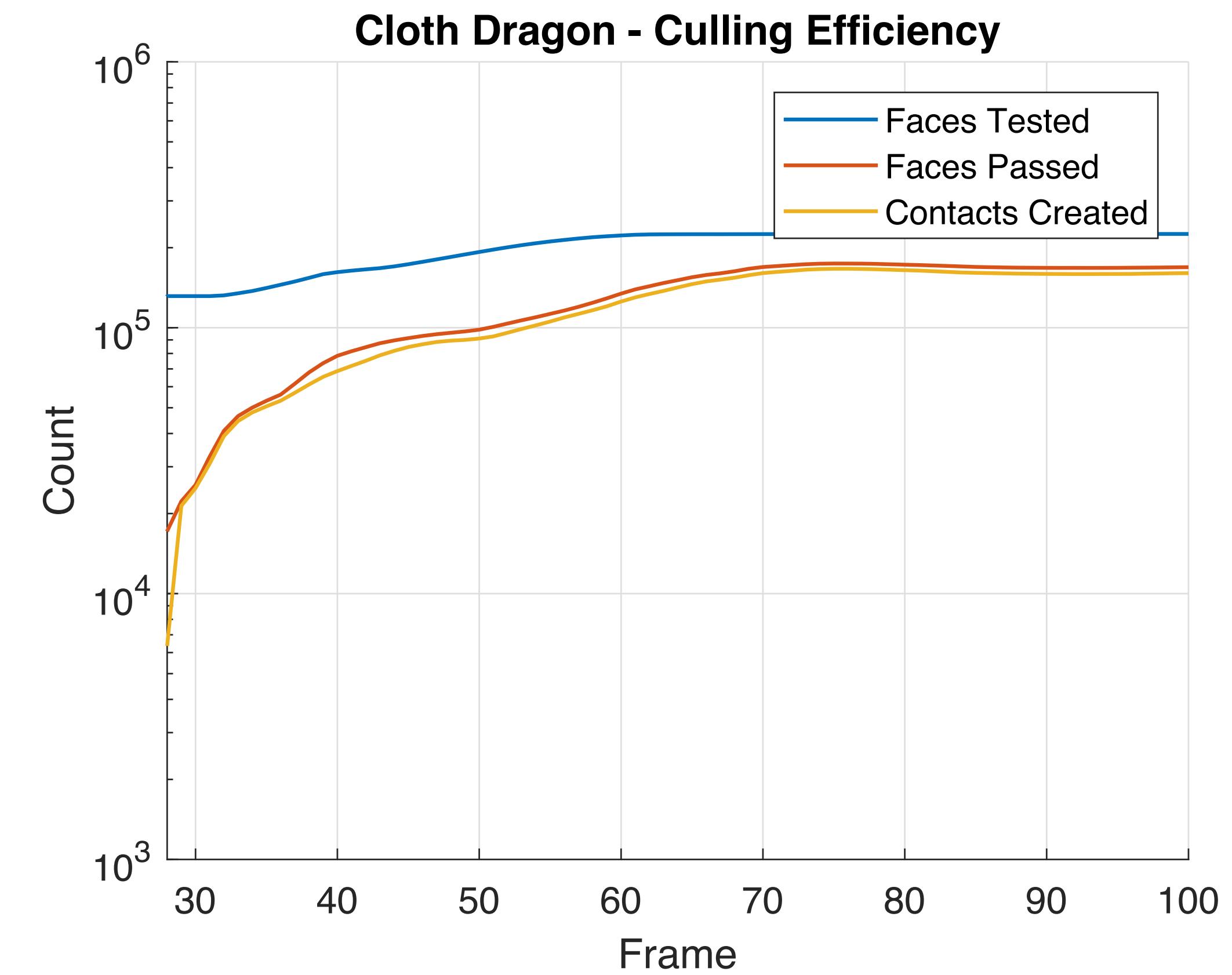
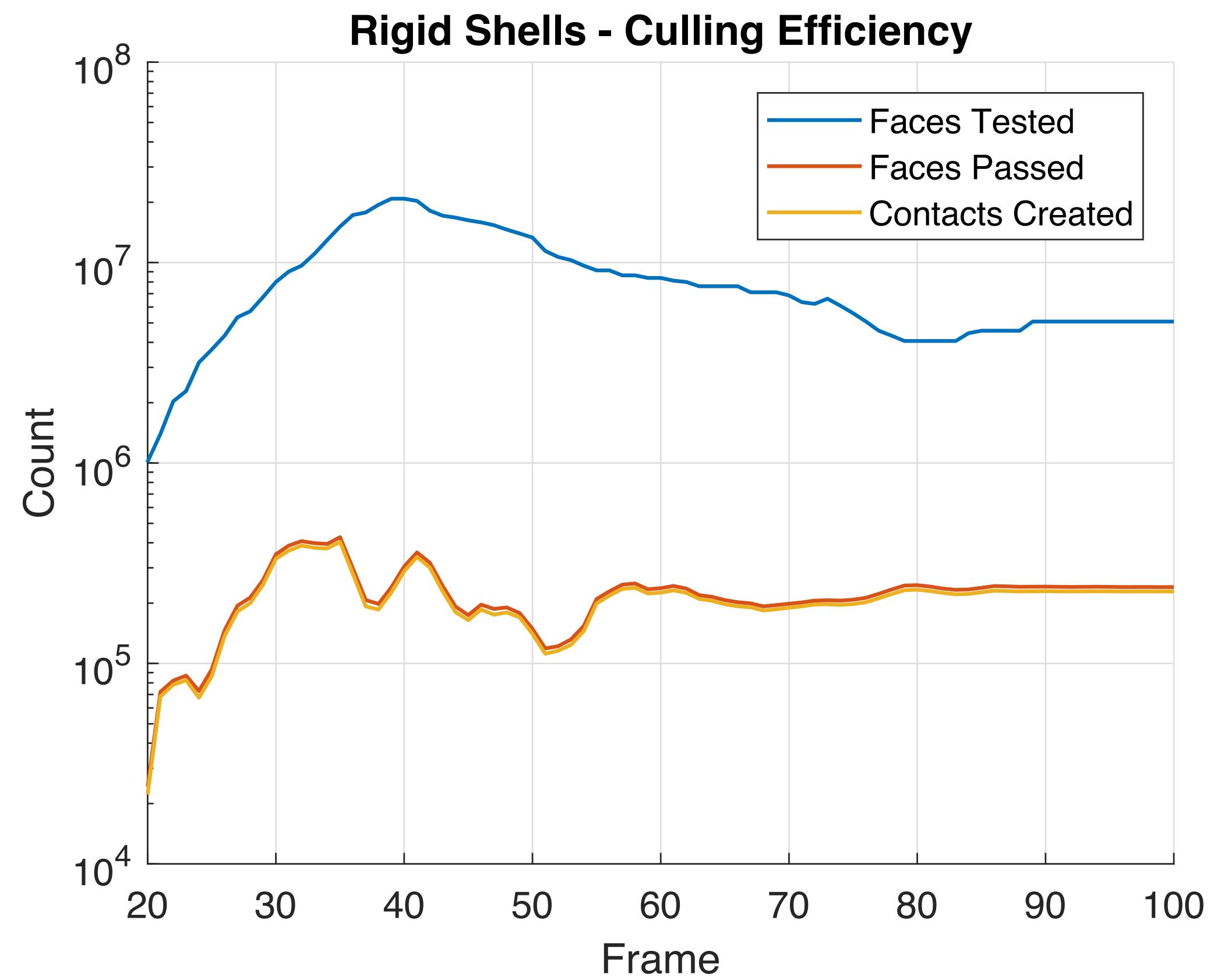
RESULTS - EDGE SAMPLED



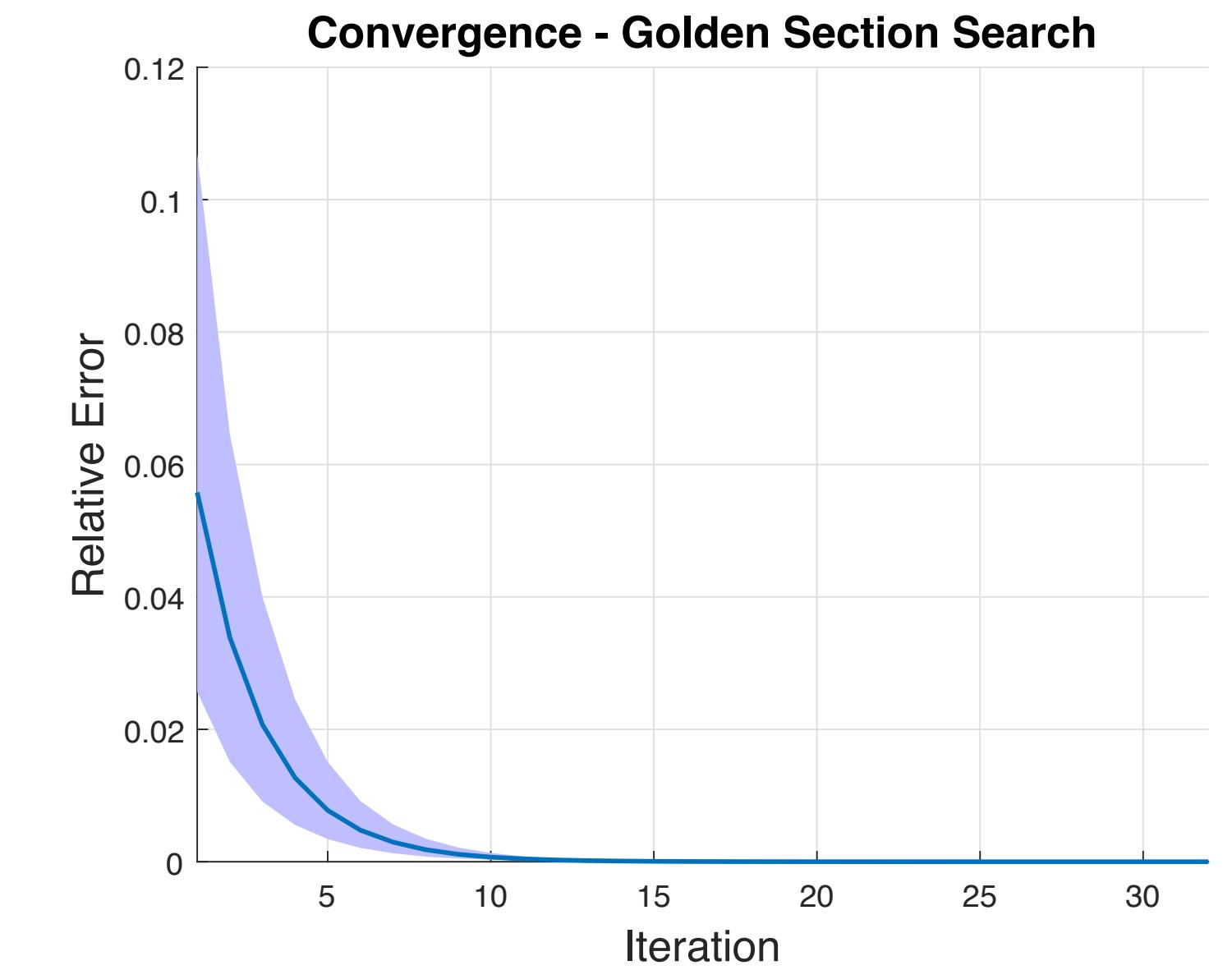
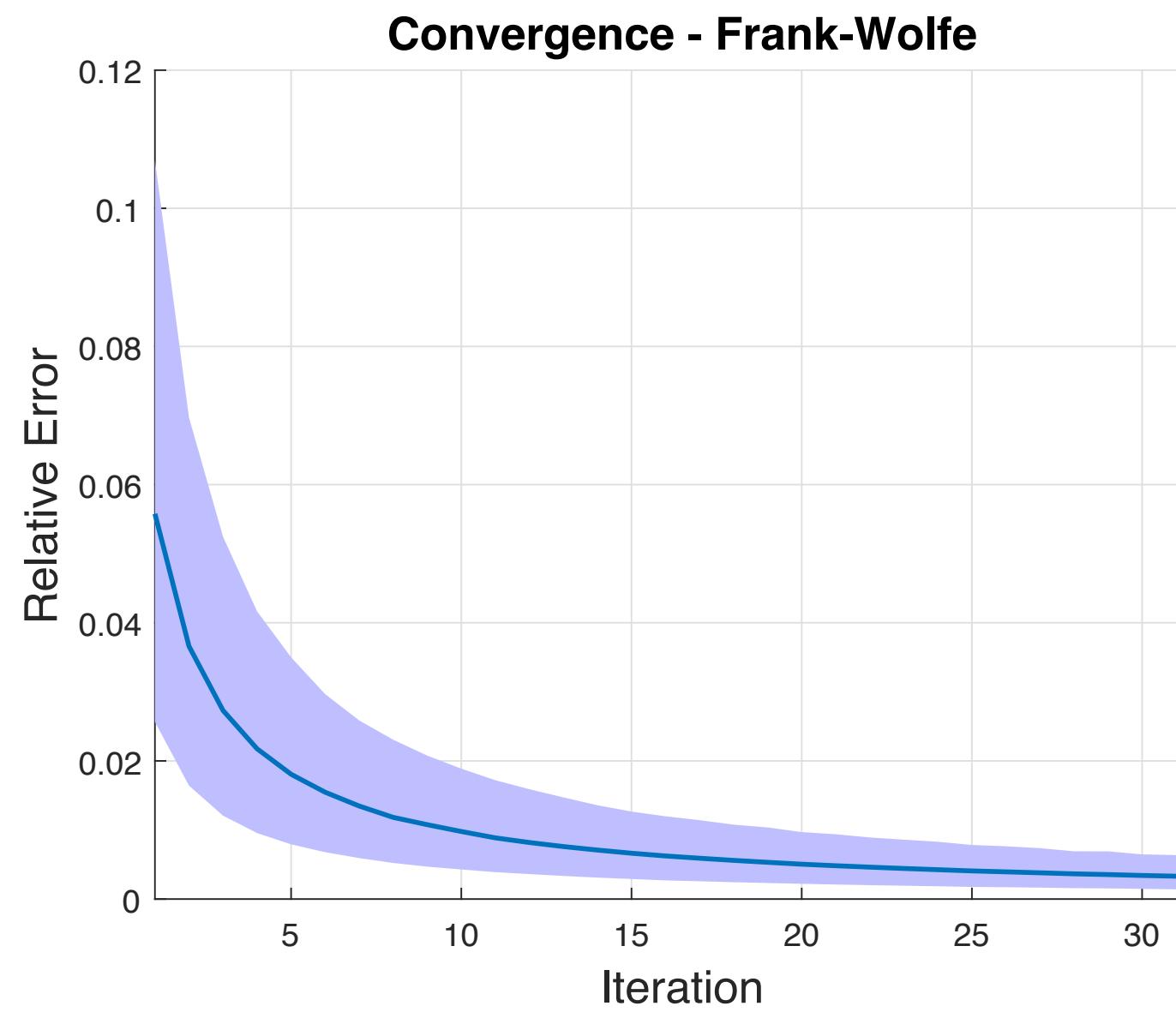
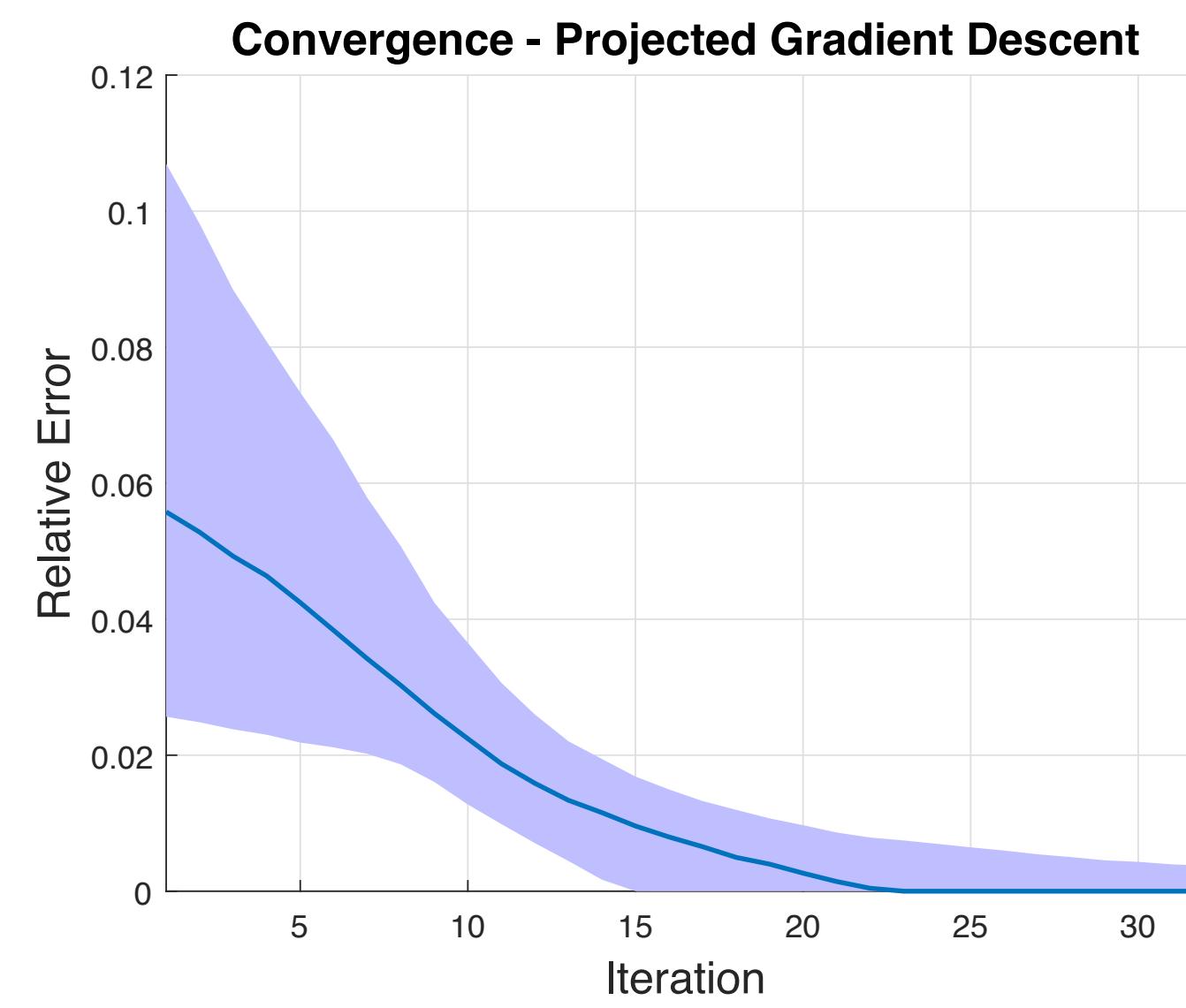
RESULTS - RIGID BODIES



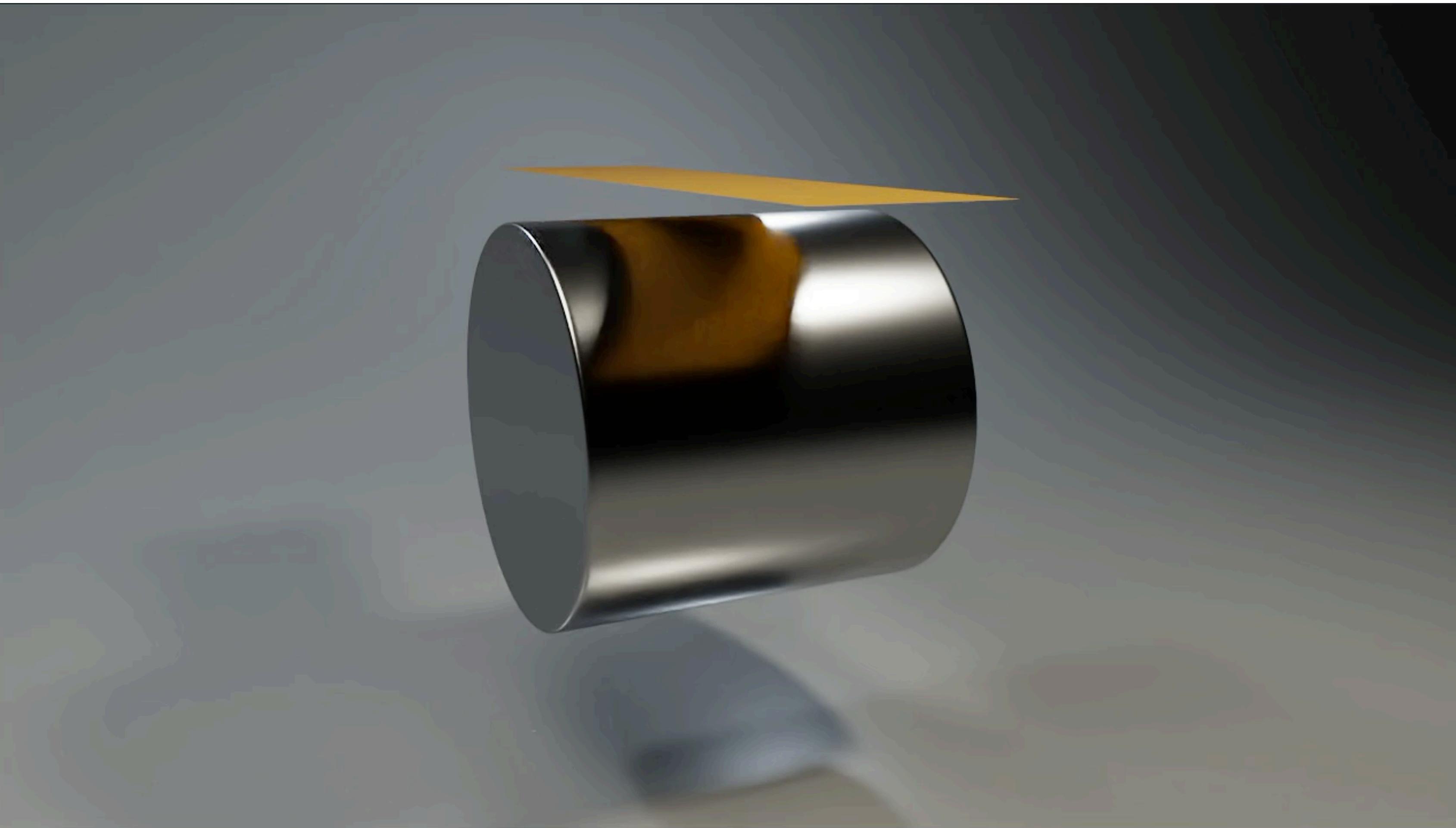
CULLING



CONVERGENCE

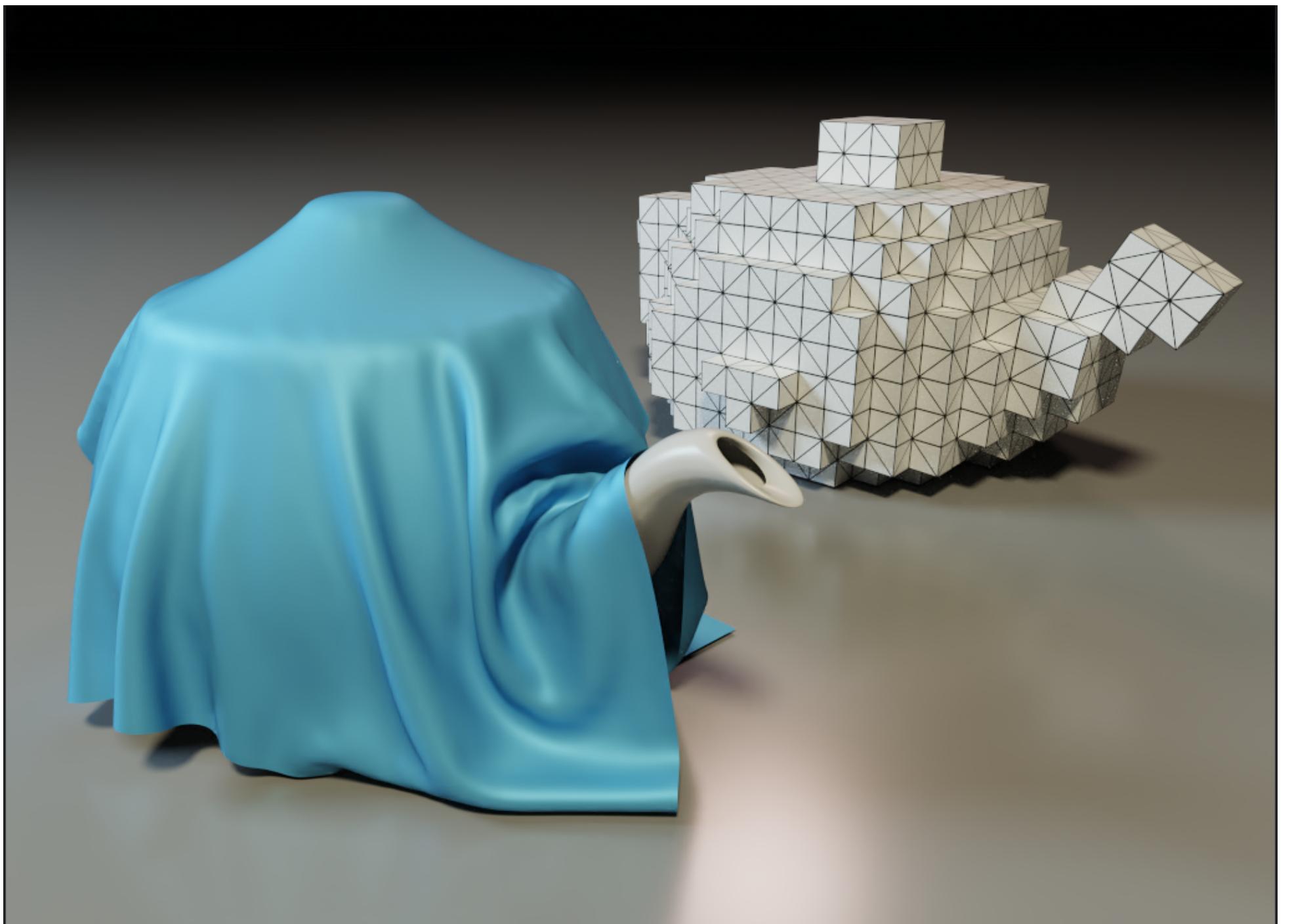


ANALYTIC DISTANCE FIELDS

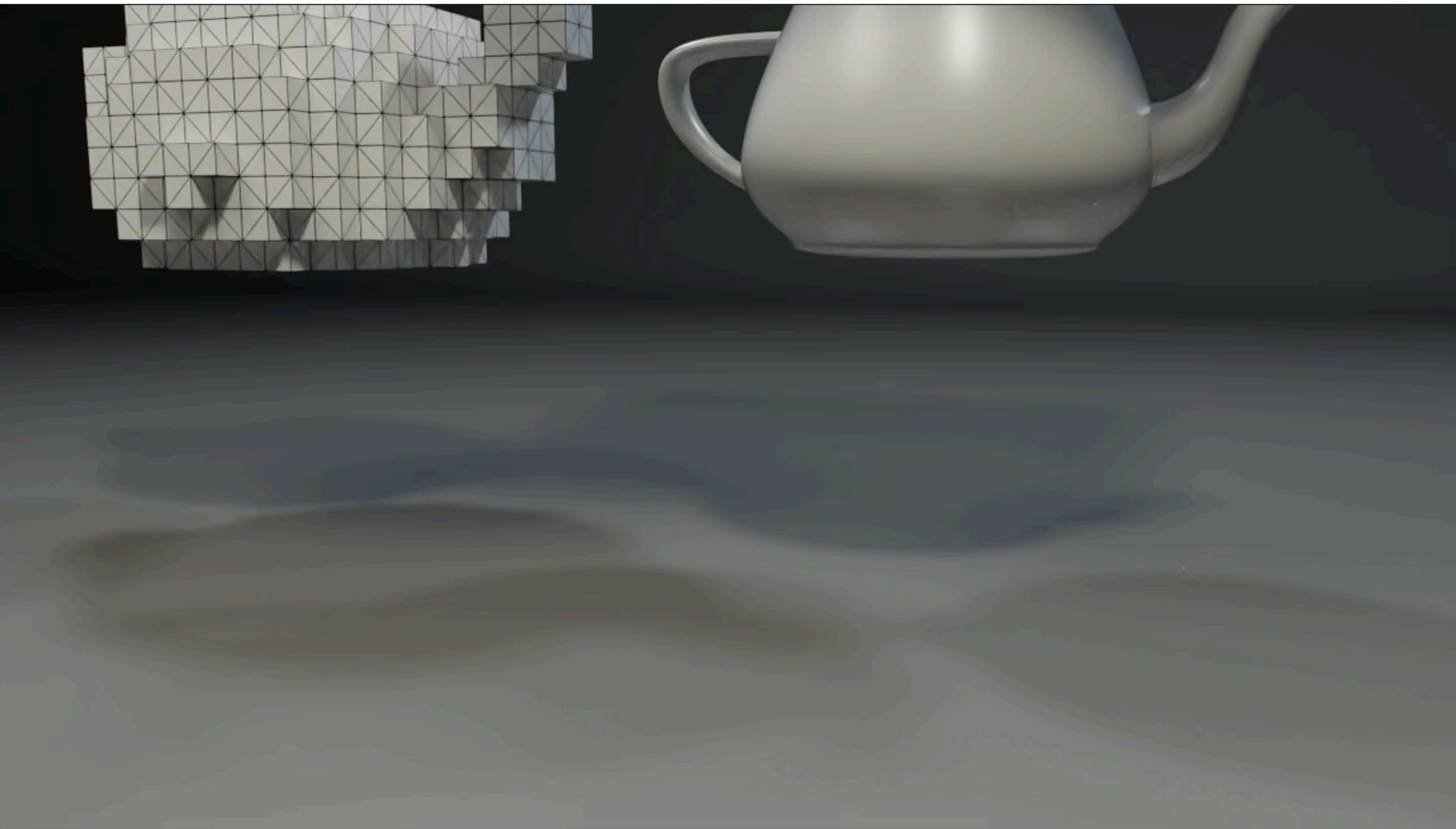


DEFORMABLE SOLIDS

- Our approach can be combined with methods to deform SDFs using cage embeddings
- [Fisher & Lin 2001, McAdams et al. 2011]
- Method:
 1. Project face to material space
 2. Perform optimization in local space
 3. Project back to world space

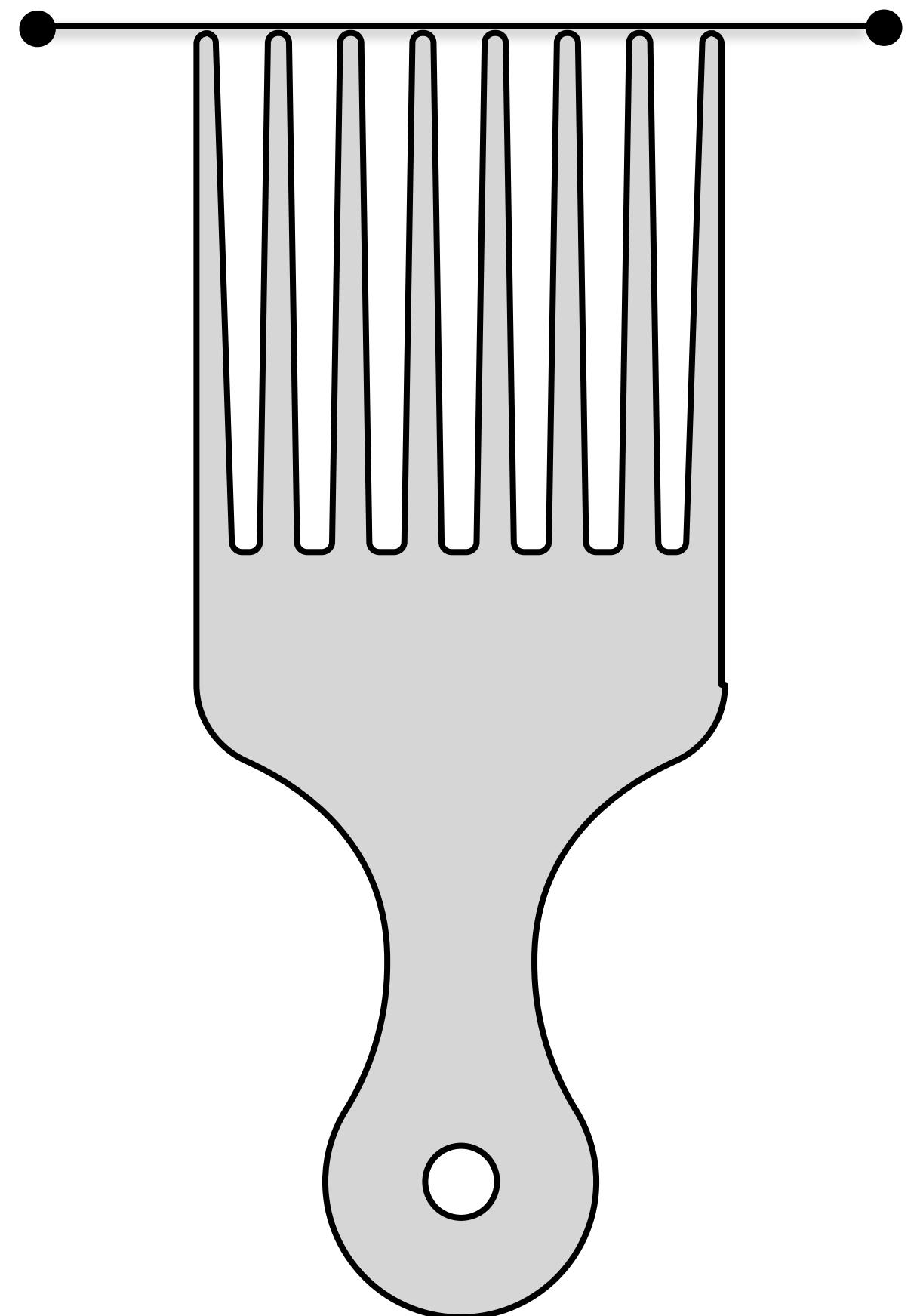
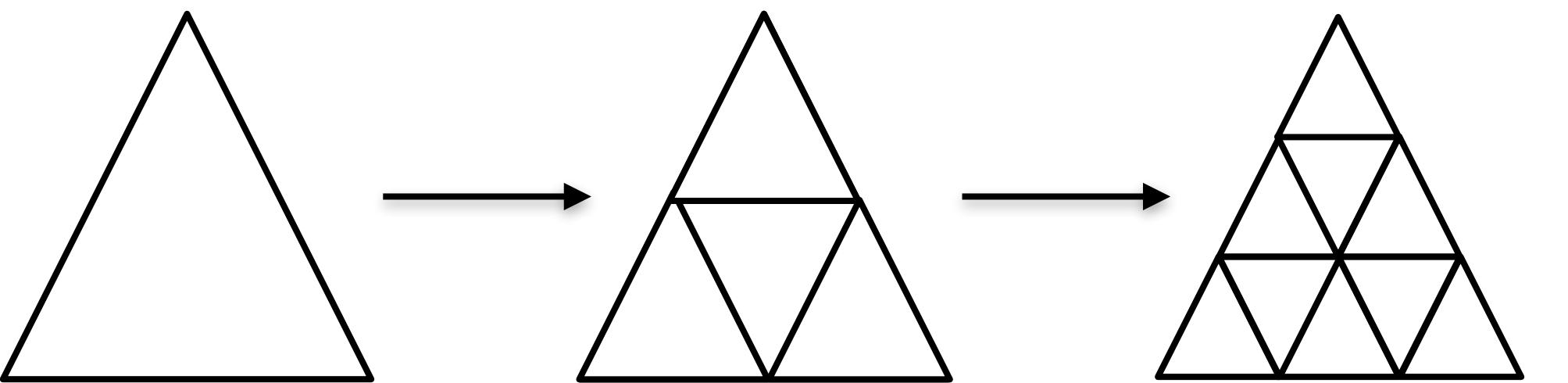


DEFORMABLE SOLIDS



FUTURE WORK

- What if we have no embedded surface representation?
- SDF/SDF contact
- Higher order schemes, e.g.: Newton, accelerated descent methods
- Implicit subdivision for multiple contacts:



REFERENCES

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